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# A NEW ENVIRONMENT TO SIMULATE THE DYNAMICS IN THE CLOSE PROXIMITY OF RUBBLE-PILE ASTEROIDS

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This paper presents a new environment to simulate close-proximity dynamics around rubble-pile asteroids. The code provides methods for modeling the asteroid's gravity field and surface through granular dynamics. It implements state-of-the-art techniques to model both gravity and contact interaction between particles: 1) mutual gravity as either direct N2 or Barnes-Hut GPU-parallel octree and 2) contact dynamics with a soft-body (force-based, smooth dynamics), hard-body (constraint-based, non-smooth dynamics), or hybrid (constraint-based with compliance and damping) approach. A very relevant feature of the code is its ability to handle complex-shaped rigid bodies and their full 6D motion. Examples of spacecraft close-proximity scenarios and their numerical simulations are shown.

## INTRODUCTION

Asteroids and comets are the current frontier of the robotic exploration of our Solar System. The motivation behind space missions to such small celestial bodies is manifold. From the scientific point of view, asteroids ad comets are invaluable witnesses of our Solar System. They constitute its fundamental building blocks and by studying them, scientists can study the history and dynamical evolution of planets and their moons, how they formed and how they evolve. Small celestial bodies represent great opportunities for spacecraft missions due to their large accessibility from Earth. They can serve as test beds for deep space demonstrations, to develop new technologies and provide suitable environment to scientific investigations. The exploration of asteroids has been identified by NASA as crucial gateway and intermediate step in the path towards the human exploration of Mars. This will critically contribute to the development of technologies required by future manned missions. In addition, asteroids and their resources play a central role in the emerging field of space economics and represent very interesting targets for new ISRU (in-situ resources utilization) concepts. Last but not least, asteroids pose a real threat to our cities and communities. In the last decades, space agencies have made investments on planetary defense, to track and characterize potentially hazardous objects, and to study mitigation strategies based on space missions. In this context, our ability to design spacecraft that can operate in the close proximity of asteroids and comets is heavily dependent on our knowledge of the properties of the celestial body and its surrounding environment.

Recent studies and in-situ exploration support the idea that a large number of asteroids in our Solar System are gravitational aggregates ('rubble piles'), as originally formulated by Chapman[1] . These bodies are made of loosely consolidated material kept together by self-gravity, and possess no to very low tensile strength. The dynamics of spacecraft in the close-proximity of such objects

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is very different compared to those around planets and their major satellites. Rubble-pile asteroids are very small and have a very weak gravity field: in such cases, self-gravity is not enough to shape the gravitational aggregate into a spherical hydrodynamic equilibrium shape. For this reason they have a very irregular shape, with uneven internal structure and mass distribution, and with a high level (up to 40%)[2] of porosity. These features rise relevant challenges to spacecraft trajectory design and planning in the close proximity of such bodies. A first area of interest concerns astrodynamics problems. A suitable and high-fidelity model of the asteroid's gravity field is required to fully and conveniently exploit natural dynamics around the rubble-pile body. Building high-fidelity models of asteroids' dynamical environment is among the current challenges of modern astrodynamics. Due to their properties, asteroids and comets possess a very weak and irregular gravity field and the dynamics in their close proximity is highly nonlinear, with a nearly chaotic behavior. A realistic model of the environment would be beneficial, in particular, to support small- to nano-satellites close proximity operations, i.e. when very limited on-board resources are available. A very timely and relevant example is that of cubesat release (e.g. AIDA[3]), or deployment of passive landers on the surface of an asteroid through ballistic descent (e.g. Hayabusa-2's MASCOT[4] and the formerly proposed Asteroid Impact Mission's MASCOT-2[5]). As for the landing problem, a second matter of concern is to predict the dynamical behavior after the interaction between the lander and the soil of the asteroid. This is always a major source of uncertainty and very few real-world data is currently available. The uncertainty at the lander-soil interaction is an extremely critical aspect in such landing scenarios and, in the worst cases, it can jeopardize the success of the landing maneuver. The problem is particularly relevant for the case of small asteroids, for which the risk of rebounding on the surface is higher. In this case the lander can unexpectedly reach escape velocity after bouncing or it can end up into not being fully operational after reaching non-nominal conditions at rest (e.g. Rosetta's Philae[6]). This contributes further to increase the complexity of such environment.

This work addresses the problem of simulating the dynamical environment in the close proximity of a rubble pile object. In addition to the complex dynamical environment due to the weak and irregular gravity field, we also consider the interaction with granular soil and dust/pebbles/boulders near the surface, as well as non-gravitational effects, including Solar Radiation Pressure (SRP). The goal of this paper is to show potential applications and in terms of scenarios to be studied. Therefore we do not provide any quantitative analysis, but only show examples of relevant scenarios. Due to the demonstrative nature of this work, parameters to simulate all examples shown are chosen such to have short simulation time, which is here the leading criterion over the realism and significance of the results.

## SIMULATING THE ASTEROID ENVIRONMENT

We present here a new software environment to host simulations of close-proximity dynamics around rubble-pile asteroids, addressing both the problem of gravity field modeling and lander-soil interaction. The implementation is based on the multi-physics Chrono::Engine C++ library[7] (also referenced as C::E) and provides additional features specifically designed to deal with the rubble-pile asteroid scenario. A more detailed description of the code can be found in the work by Ferrari et al[8] .

The mass distribution and enveloping shape of the rubble-pile body is found after simulating its formation as a gravitational aggregate of boulders and pebbles (also referenced as particles). To support this goal, the software features state-of-the-art techniques to reproduce gravity and contact interaction between particles. In the followings, we provide details on the numerical methods and algorithms available to solve gravitational and contact dynamics ad we discuss the capabilities of the code, by detailing its most relevant features and performance. We show examples of potential applications related to close-proximity rubble-pile scenarios. These include gravitational dynamics with perturbations due to SRP close to the surface and bouncing/hopping dynamics after landing (lander/soil interaction modeling).

## Granular dynamics

The ability to simulate granular dynamics is crucial when dealing with a high number of boulders and pebbles that interact under their mutual gravitational attraction or due to external actions (e.g. lander-soil interaction). Typically, two classes of methods are used to solve for contact interactions: hard- and soft-body methods. Hard-body methods consider infinitely rigid bodies and instantaneous collisions between them[9,10] . These are constraint-based numerical methods and are typically very stable and suitable to simulate the non-smooth dynamics. Conversely, soft-body methods consider the finite rigidity of the bodies at contact and model the elastic behavior of the material[11] . Unlike hard-body methods, these are force-based methods and can reproduce more accurately smooth dynamics. Also, they are needed to simulate phenomena such as wave propagation in granular media,[12] where the assumption of infinitely rigid body would lead to unrealistic results. Because of a more detailed modeling of contact dynamics, soft-body methods are in general more realistic than hard-body models. However, the increased accuracy is obtained at the cost of a lower numerical stability and higher computational effort. In fact, in order to accurately reproduce the elastic behavior of the material, the time step of the simulation must be extremely small, to adequately sample the high-frequency dynamics due to the stiffness of the material. I this case, a constraint on the time step $\delta t$ to be used can be derived by computing the characteristic time $T_k$ of a spring-dashpot system[13] :

$$\delta t_k < \frac{T_k}{2} = \frac{\pi}{\omega_k} \tag{1}$$

with

$$T_k = \frac{2\pi}{\omega_k} = 2\pi\sqrt{\frac{m_r}{k}} \tag{2}$$

and where $\omega_k$ is the highest frequency in the system of bodies, i.e. considering the highest stiffness $k$ and lowest reduced mass $m_r$*. Practical applications often make use of constraints even more restrictive than (1), ranging from $T_k/10$ to $T_k/\pi$,[14] or even smaller than $T_k/100$.[15] To summarize, both classes of models have their own pros and cons and their use must be weighted upon the application scenario. The identification of the method to use is not always an easy task: contact dynamics are not supported by an exact analytical formulation and rely on several parameters, which are not directly measurable and often offer poor physical interpretation.[16] However, these represent a powerful tool to study the complex granular phenomena and have found many applications in the past, not only to simulate rubble-pile scenarios,[14,17–19] but also planetary ring dynamics,[20] cratering formation[21] and many others.

A major concern regarding the realism of granular simulations is about the shape of interacting particles. All aforementioned codes make use of spheres: however, this is a very relevant simplification on particles' shape, which can lead to very inaccurate results,[22] as shown in experimental results of granular dynamics. In fact, spheres are not suitable to reproduce effects typical of angular

*$m_r$ is the reduced mass of the two least massive bodies in the system

bodies, such as interlocking. Few attempts have been made to use angular shapes instead of spheres. Movshovitz et al[23] use game engines to solve for contact interactions, but the computational accuracy provided by such engines is typically very low (single precision). Ferrari et al[8] use convex hulls in a code implemented using C::E,[24] providing promising double precision results.

Our code builds upon the work by Ferrari et al.[8] Interacting bodies are treated as rigid bodies, with both center of mass and rotational motion (6 DOF). They can be of any shape to be input by the user (triangulated mesh). In addition, ready-to-use shapes include geometrical shapes (sphere, box, cone, ...) on randomly-generated convex hulls (given their number of vertices). The code is integrated with C::E, which offers three options to solve for contact interactions:

- NSC (Non-Smooth Contact). It is a constraint-based hard-body method. The equations of motion are formulated in terms of Differential Variational Inequalities (DVI) and require a solution of a Cone Complementarity Problem (CCP) at each time step. Contact dynamics are modeled through impulsive collisions and the colliding bodies exchange momentum instantaneously, based on the coefficient of restitution. This parameter represents the amount of dissipation due to the collision and it can be set by the user between 0 (fully inelastic collision) and 1 (fully elastic collision). The NSC is best suited to non-smooth problems, with discontinuities or with nearly rigid contacts.[25–27]

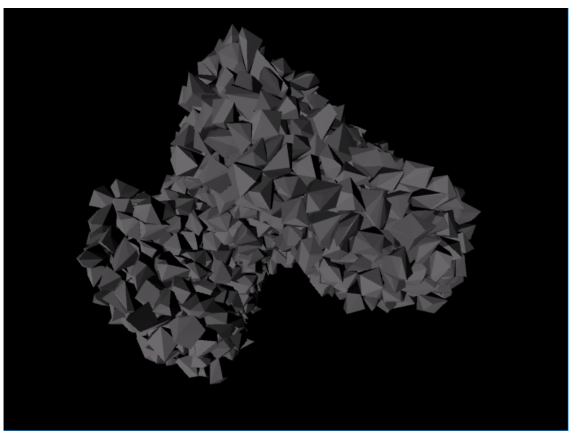
- SMC (SMooth Contact). It is a penalty-based soft-body method. The equations of motion are formulated in terms of Differential Algebraic Equations (DAE), as a system of Ordinary Differential Equations (ODE, the dynamics) + Algebraic Equations (AE, the geometrical constraints). Contact dynamics are modeled using a spring-dashpot system, whose parameters can be tuned by the user (stiffness and damping). The collision occurs in a finite time and the method is best suited for smooth problems.[28]

- NSC with compliance and damping. Its formulation is the same as NSC, but the constraint is enforced with compliance and damping. This method can be used both for non-smooth and for smooth problems, since it can reproduce the viscoelastic behavior of the material, with less stringent constraints on the time step with respect to the SMC method.[29] This method is able to joint the advantages of both hard-body methods in terms of numerical stability and of soft-body models in terms of accuracy of the dynamics. It overcomes the classical problem of endless bouncing, when two bodies never reach a rest condition after they get in contact. This is typical of hard-body models and it is due to the rigid enforcement of the 'contact/no-contact' condition.

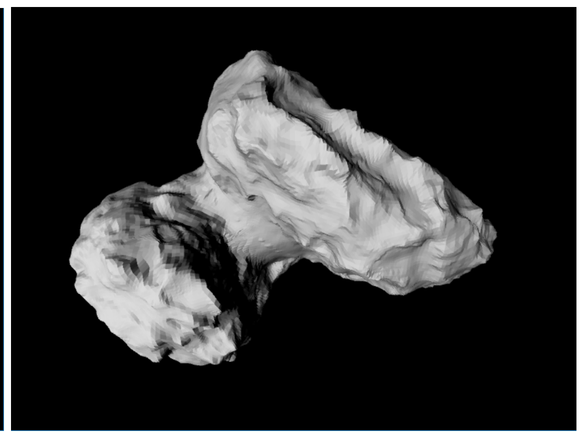
**Creating the mass model**

In this section we show examples of rubble-pile mass models for given small bodies, generated using our code. When dealing with the gravity field in the proximity of a non-spherical mass distributed source, classical methods make use of spherical harmonics series to model the gravity potential around the celestial body. However, to be accurate, these method requires a lot of information, typically available through close-proximity gravimetry and largely not available for the case of asteroids, whose known data relies mainly on remote radar or optical observations. Also, the spherical harmonics series does not converge inside the Brillouin sphere of the source object and thus it is not appropriate to model the dynamics close to the surface of oddly-shaped asteroids[30] . In recent years, strategies have been developed to provide shape-based estimates of the gravity field around asteroids. The potential of elongated asteroids can be represented using ellipsoids[31] or, if a shape model is available, refined using a polyhedron model[32] . The latter represents the state-of-the-art technique to reproduce the potential of a complex-shaped body. However, shape-based models consider the interior of the body to be homogeneous. In general, they are not suitable to represent uneven mass distributions, including internal voids and porosity. As mentioned, these are typical features of rubble-pile asteroids, and can be modeled using mass concentrated (also called 'mascon') models.[33,34] Such models entail the use of many concentrated masses to reproduce in a discrete way the mass distribution of the body and consequently, its exterior gravity field as the sum of many central fields. Mascon models are highly tunable and versatile, but the resulting field depends on many parameters (value and location of concentrated masses). To be accurate, these models often require a real-world reference, which is not always available for the case of asteroids.

As first example, the rubble-pile object is obtained through gravitational aggregation of smaller bodies and particles (boulders and pebbles). The gravity model obtained is treated as a mass-concentrated (mascon) model where the pattern of concentrated masses is obtained after a natural process of self-aggregation. In our code, mutual gravity between the N bodies can be enforced through two different options: (a) direct N-to-N calculation between all bodies or (b) Barnes-Hut[35] octree GPU-parallel implementation[36] . From the mutual gravity interaction point of view, each particle is treated as a point mass. Here we show examples of full rubble-pile body and rubble pile with monolithic core.



(a)



(b)

**Figure 1. (a) Full rubble-pile model of comet 67/P Churyumov-Gerasimenko made of 2270 convex hulls. (b) Shape model used to extract the bodies from a parent aggregate of 5000 bodies.**

*Full rubble-pile.* Two different strategies can be followed to obtain a full rubble pile object. The fist strategy consists in creating a rubble-pile object of a given shape. In this case, we proceed with a two-step procedure:

- First, we generate a large number of bodies with properties given by the user. These include the shape of each body, their size distribution and their inertial properties. We show here an example using convex hulls. Each convex hull is generated as the enveloping surface of a cloud of randomly generated points. The user can choose the number of points (which sets the maximum number of vertices of the hull: for 15 points we have hulls with 8-12 vertices) and their spatial distribution (which sets the characteristic size of the body and its elongation, by means of its axial ratios). The user can also set the surface properties of the bodies (friction coefficients), the contact model (NSC, SMC or NSC w/compliance) and its parameters (restitution coefficient or stiffness and damping). Finally, the numerical simulation is set up by selecting the numerical integrator/solver and the time step. After the N bodies are generated in the domain, we let them interact and aggregate through mutual gravity and contact dynamics. In this phase, we are only interested in creating a stable and large aggregate of bodies (also called *parent* aggregate) and we are not interested in the transient dynamics: therefore we select NSC and high time steps (on the order of tens of seconds).

- Once we obtained the parent aggregate of particles, we extract from it a selection of particle such to form a given shape, provided as a triangulated mesh. For each particle in the large aggregate, we check whether it is inside or outside the triangulated mesh. Only bodies internal to the wanted shape are kept to form the rubble-pile object. An example is shown in Figure 1, for the case of comet 67/P Churyumov-Gerasimenko. In this case, the rubble pile model of the comet is made of 2270 bodies of equal characteristic size, obtained from a parent aggregate of 5000 convex hulls.

Alternatively, in case the final shape of the object is not known, the rubble pile model can be generated as an equilibrium shape, after the gravitational aggregation process. In this case, the dynamics of the initially created bodies must be set carefully, as well as their surface and contact dynamics. Each set of kinematic conditions (initial linear and angular position and velocity of each body), surface properties (friction) and contact properties (model, parameters) will generate a different equilibrium state after aggregation and, consequently, a differently stable aggregated shape.

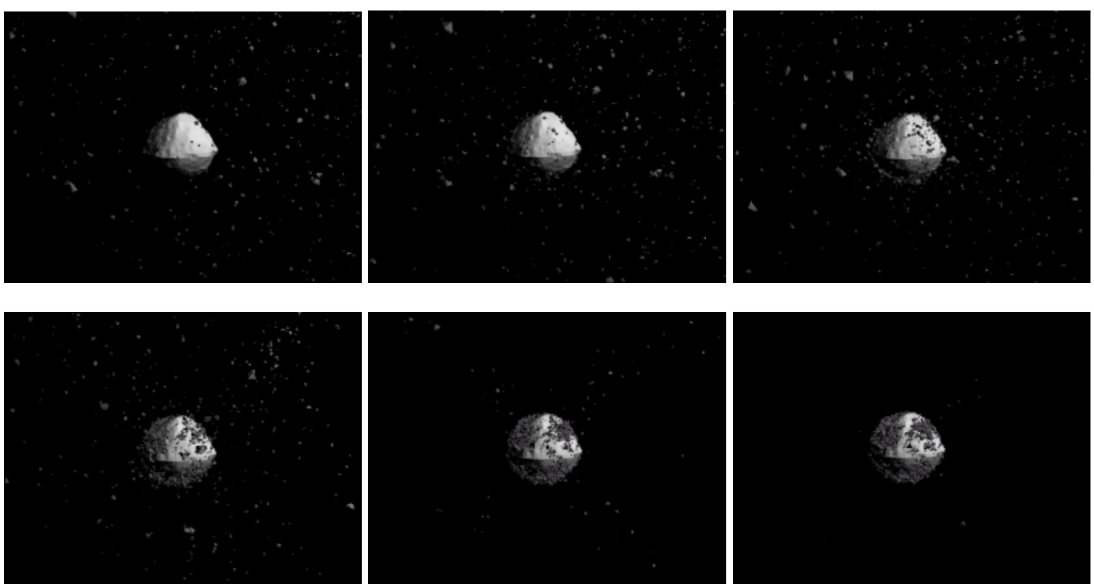
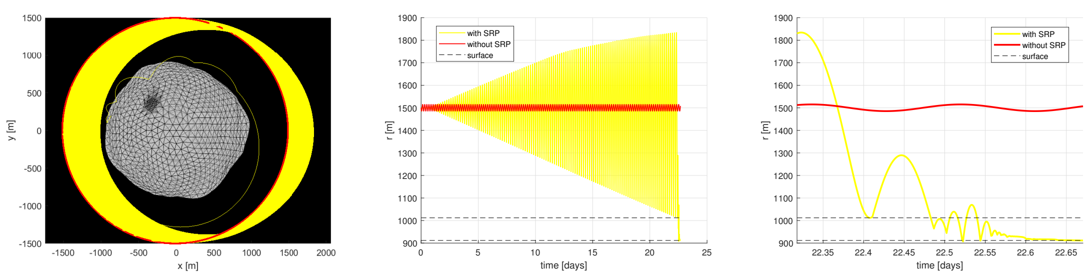


**Figure 2. Model of asteroid 101955 Bennu: central core (traingulated mesh) + layer of granular material on the surface (1942 bodies)**

*Monolithic core.* In some cases, generating a full rubble-pile object can be computationally very intensive, especially if the characteristic size of the particles is very small compared to the size of the full aggregate. Also, for some applications, it might not be needed to have a full rubble-pile, but also to consider the granular nature of the body in some confined regions of the domain. In these cases, we can build the mass model of the asteroid by a combination of large monolithic blocks and smaller granular aggregates. We show here an example of a model of asteroid 101955 Bennu: we use its triangulated shape model as a central core and we add a layer of granular material on its surface, through N-body gravitational simulation, as shown in Figure 2.

### Non-gravitational forces

In the context of simulating the dynamics of space probes in the close-proximity of asteroids, the user can set custom forces other then gravity, to act on the spacecraft. One example is that of Solar Radiation Pressure (SRP), which is very relevant when considering the environment around small celestial bodies. Due to the extremely low gravity field of such bodies, the magnitude of the acceleration generated by the SRP on a spacecraft is often comparable and not negligible. Also, the SRP must be taken into account when simulating the complex dusty environment near the surface of the asteroid. Our code can be used to simulate the dynamics of boulders and pebbles very close to the surface and their evolution in time. Figure 3 shows the results of the simulation of the dynamics of a boulder of about 70 m in size, orbiting very closely around asteroid 101955 Bennu. Figure 3(a) shows the evolution of the boulder's trajectory due to the action of SRP (Sun is towards positive *y* direction), while Figure 3(b) shows the evolution in time of its distance from the center of Bennu and shows a comparison between a model with (yellow) and without (red) the effect of SRP. As expected, the SRP has the overall effect of increasing the eccentricity of the orbit by increasing its apocenter and decreasing its pericenter, up to a point when the boulder hits the surface and, eventually comes to a rest on it. Figure 3(c) is a magnification of the last phase of the evolution shown in Figure 3(b). The dotted lines show the limiting boundaries of the surface of Bennu in terms of distance from its center. The boulder is shown to bounce several times on the surface before reaching its final position at rest. The bouncing dynamics is modeled using the granular layer of particles over the central monolithic core of Bennu, thus increasing the realism of the numerical simulation.

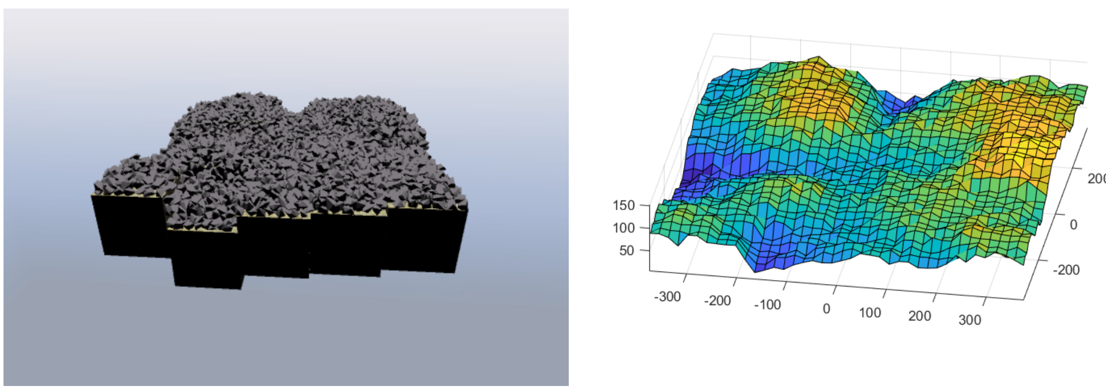



(a) (b) (c)

**Figure 3. Evolution of a 70 m boulder in the close proximity of asteroid 101955 Bennu. The effect of SRP is highlighted (yellow trajectory) and compared to its dynamics without SRP (red trajectory). The Sun is towards the positive y direction (a)**

### Interaction with terrain

The final phase of the evolution in Figure 3 shows the interaction between a boulder and the surface of asteroid Bennu. In this case, the initial orbital condition is such that, after hitting the surface for the first time, the boulder has enough energy to bounce multiple times and fly around the asteroid before reaching its final position at rest. However, most of the times, scenarios of interactions with the soil affect only a small portion of the surface. In these cases, to optimize numerical performance and computational time, it is best to generate only the limited portion of the surface affected. Figure 4 shows an example of a granular terrain patch, generated using our code. The surface terrain is generated such to have random slopes, hills and valleys. The terrain is created as a thick layer of particles on a rigid floor. In particular, we create the particles above a flat floor made of blocks and we let the particles settle under a downwards constant gravity. After the granular layer has reached a steady state condition, we move downwards the different blocks very slowly, with constant velocity. Each block has a different velocity, generated randomly. Due to the motion of the blocks, the upper granular layer slowly adjust according to friction between the bodies and contact parameters chosen by the user. After a given time, the blocks stop their motion and the terrain settles to a new, non-flat configuration, with hills and valleys, as shown in Figure 4(a). Figure 4(b) shows in a clearer way the morphology and slopes of the upper surface generated through this process.




(a) (b)

**Figure 4. Granular terrain patch with randomly generated slopes, hills and valleys on the upper surface**

### Touch down and hopping

Terrain patches can be used to simulate different soil interaction scenarios. In the example shown here, we consider a lander-soil interaction. The goal of such study could be to investigate the effects of the many contact-related parameters involved and to identify those playing a major role. Snapshots from a sample demo simulation of a cubesat-sized lander on granular soil are shown in Figure 5. A similar scenario is that of hopping dynamics. In this case, the simulation starts with the cube-shaped hopper probe on the surface at rest. The hopper is instantaneously provided with a non-zero angular velocity, to simulate the effect of internal wheeled actuation. Its bouncing dynamics sequence is shown in Figure 6.

## CONCLUSION

The paper shortly describe the capabilities of our code in terms of methods and algorithms available to solve gravitational and contact dynamics. In the context of close proximity dynamics around

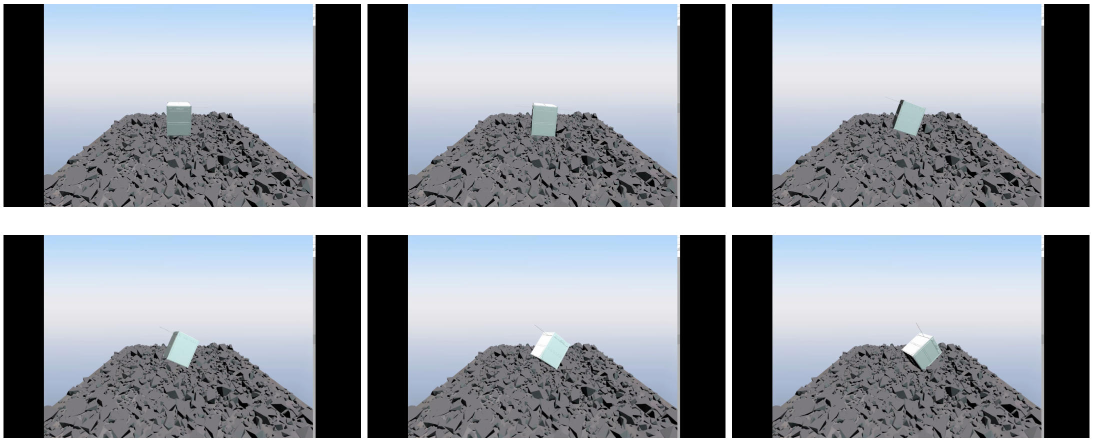


**Figure 5. Touch-down phase of a landing sequence for a sample cubesat-sized lander on a granular soil. Cubesat's '.obj' model taken from NASA 3D Resources website (`https://nasa3d.arc.nasa.gov/models`)**

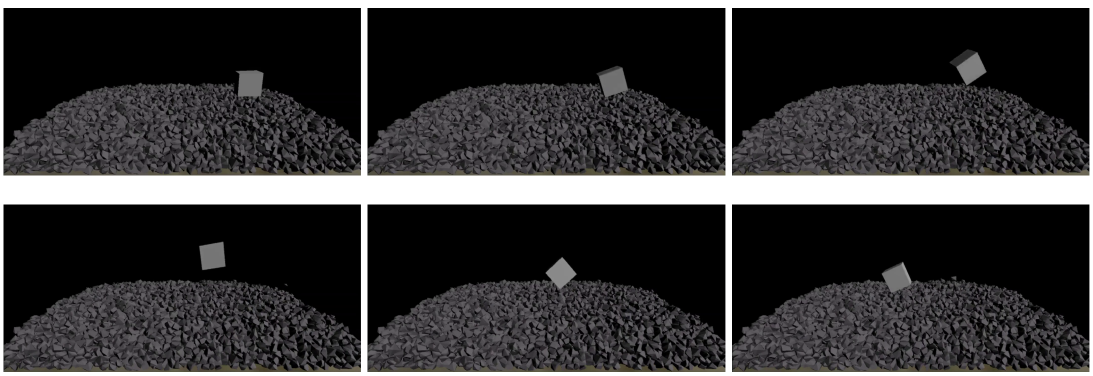


**Figure 6. Hopping sequence of a cube-shaped hopper probe on a granular bed of 10000 particles**

small celestial bodies, we show potential applications for our code. In particular, we show applications of gravitational aggregation to create mass distribution models for rubble-pile objects and to simulate the complex and dusty environment near their surface, considering the dynamics of boulders and pebbles. In addition, we show applications related to the interaction with the soil of such objects and show examples of granular terrain patches generation, lander and hopper dynamics. As mentioned, all simulations performed in this work are purely demonstrative and can be performed in a few hours with a common medium-range laptop. However, the good numerical performance of the simulations in the example shown hints a good capability of the code to reproduce more complex scenarios, including a higher number of bodies.

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